



# Centralized netting in financial networks<sup>☆</sup>

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## ABSTRACT

We consider how the introduction of centralized netting in financial networks affects total netted exposures between counterparties. In some cases there is a trade-off: centralized netting increases the expectation of net exposures, but reduces the variance. We show that the set of networks for which expected net exposures decreases is a strict subset of those for which the variance decreases, so the trade-off can only be in one direction. For some network structures, introducing centralized netting is never beneficial to dealers unless sufficient weight is placed on reductions in variance. This may explain why, in the absence of regulation, traders in a derivatives network do not develop central clearing. Our results can be used to estimate margin requirements and counterparty risk in financial networks. We also provide techniques to evaluate the robustness of our results to behavioral responses to the introduction of centralized netting.

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## 1. Introduction

Centralized netting in a financial network is the novation of exposures to a single counterparty. This can reduce the aggregate level of exposures in the system by netting offsetting claims, and so decrease systemic risk and collateral requirements. Examples of centralized netting include the introduction of a central counterparty (CCP) to over-the-counter derivatives markets, triparty repo, and netting in payments networks.<sup>1</sup> Improvements are most likely when all exposures are simultaneously netted.<sup>2</sup> Introducing centralized netting in a subset of exposures has the effect of improving netting amongst those exposures, because they are now novated to the same counterparty. But it disrupts bilateral netting sets amongst those exposures that are not novated. This paper ex-

amines how this trade-off depends on the structure of the network.

We focus on the introduction of a central counterparty (CCP) to a derivatives network. Duffie and Zhu (2011) show that, when a single asset class is centrally cleared, bilateral exposures between agents may increase — as a result of reduced netting opportunities across pairs of counterparties — resulting in an overall loss in netting efficiency. They derive conditions, in terms of the number of asset classes and the number of dealers, for whether or not centralized netting in the form of a CCP is beneficial.

In Duffie and Zhu's framework, all agents in the network are assumed to trade with one another. By expressing the magnitude of links between agents in each asset class as a random variable, they can then calculate expected exposures with respect to this distribution, and examine how this changes before and after the introduction of centralized netting. A clear advantage of this approach is that it is not necessary to observe the actual network exposures; instead it relates the question of whether or not the introduction of centralized netting in a single asset class is beneficial or not to easily observable parameters. However, one drawback to their analysis is the assumption of a completely-connected network, with all agents trading with one another. Typically, real-world financial networks are not completely connected. Empirical studies show that there is typically a number of well-connected counterparties coupled with a larger number of more poorly-connected counterparties.

In this paper we develop a model of centralized netting that extends the analytic framework developed by Duffie and Zhu to more

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<sup>1</sup> For example, CLS (Continuous Linked Settlement) nets foreign exchange transactions in payments systems.

<sup>2</sup> This may not be the case when the agents strategically reform the network in response to the introduction of centralized netting. See Section 6.

general network models. We develop analytical results that do not depend on any assumptions about the precise structure of the network. In addition, we examine the effect of centralized netting on the variance, as well as the expectation, of netted exposures. Our rationale is that, if the agents have some aversion to volatility or extreme outcomes, then they are likely to benefit from a lower variance of net exposures, as well as a lower mean. As in Duffie and Zhu, we show that the introduction of centralized netting is more likely to be beneficial when there are more agents, and when there are fewer asset classes.

Our first key result is that, for any given network, if introducing centralized netting reduces expected exposures, then it must also reduce the variance of exposures. However, the converse implication is not true. This means that the set of networks for which the expectation of exposures is reduced is a subset of those for which the variance is reduced.

We derive general expressions to determine cases when centralized netting reduces the expectation and variance of exposures. These expressions are in terms of the network distribution and the number of asset classes. As the benefits of centralized netting are decreasing in the number of asset classes in the network, we can determine the critical number of asset classes above which centralized netting does not deliver benefits. This critical number is higher for the variance than for the expectation of exposures.

These first results require full information about the structure of the network. Our second result weakens this assumption. We establish upper and lower bounds on the maximum number of asset classes required for centralized netting to deliver benefits. These do not depend on knowing the degree distribution of the network. This second result could help a decision maker to judge the benefits of central clearing when the full structure of the network is unknown. We show that the Duffie and Zhu network structure achieves the upper bound, suggesting that their criterion is extreme and will lead to central clearing being introduced too often.

Our additional results relate to specific network structures. The previous literature has demonstrated that real-world financial networks can be accurately modeled as scale-free network models (e.g. (Soramäki et al., 2007; Inaoka et al., 2004; Garlaschelli et al., 2005)) or by core-periphery models (e.g. (Craig and von Peter, 2014; Langfield et al., 2014; Markose, 2012)). For scale-free networks, we find that expected net exposures *always* increase when a single asset is novated to a CCP, regardless of the size of the network. Therefore, for the introduction of central clearing to be a optimal policy, dealer agents or policymakers must have some desire to reduce the variance of net exposures. Moreover, for the asymptotic network we find that, when a single asset is novated to a CCP, expected net exposures always increase and the variance of net exposures always decreases. Hence, the trade-off between mean and variance is a definite feature of the limiting scale-free network. In contrast, for core-periphery networks we find that, for any given number of asset classes, there is a minimum size of the network above which the introduction of a CCP reduces both the mean and variance of net exposures. Thus, for sufficiently large core-periphery networks, centralized netting is unambiguously beneficial.

Our findings can be applied to predict the impact of introducing a CCP on margin requirements, since aggregate margin needs are related to the distribution of netted exposures.<sup>3</sup> Our model can also be applied to any network where the issue of bilateral vs centralized netting is under consideration. For example, it could be used to consider the effect of multilateral netting in the interbank market, the benefits of bilateral vs triparty repo, or the effect

on payment system exposures of introducing a netting mechanism (such as CLS or a liquidity-saving mechanism).

The techniques we develop in this paper can be used to evaluate the efficiency gains of novating multiple asset classes to a single CCP or of different configurations of CCPs that each handle different sets of asset classes. Trivially, if it is efficient to novate one asset class to a CCP, then it will always be efficient to novate another. This follows from the fact that efficiency gains to novation are higher when the number of asset classes is lower and novating an asset effectively reduces the number of asset classes by 1. The logical extension of this idea is that the well-known result that the unconstrained first-best is to put everything through a single CCP. Relatedly, we show that (1) merging CCPs will always improve efficiency, again with the ultimate conclusion that a single CCP is best, and (2) it is most efficient to novate asset classes to the largest existing CCP.

We also consider how the results may change if the agents strategically reform the network in response to the introduction of centralized netting. We show that, in certain circumstances, centralized netting may be more efficient when there are *more* asset classes, rather than fewer. This can happen if the post-clearing network is less connected, perhaps as a result of agents raising their bilateral risk limits as a result of the new system infrastructure, and thus needing to form fewer links. Bilateral netting becomes more efficient when there are fewer connections, so it is more likely that the CCP will be beneficial when it disrupts a smaller fraction of the bilateral links.

The final two sections discuss how our results can be used in practice, and the implications of our findings for policy. The issue of clearing networks is likely to come to the fore in Europe in the near future as the European Commission is considering requiring the clearing of euro-denominated swaps to be carried out within the European Union. Most such clearing is done in the United Kingdom, which has announced its intention to leave the European Union in March 2019. Our model allows analysts to forecast the disruption to netting in clearing networks that may result.

## 2. A review of the relevant literature

The framework developed in Duffie and Zhu (2011) has been utilized by other authors in order to investigate specific problems. Heath et al. (2013) is perhaps the most similar to our paper in that they examine a network other than the completely-connected structure of Duffie and Zhu. They assume a core-periphery structure and use a computational approach to compare the effect of various clearing arrangements on expected netted exposures.

Anderson et al. (2013) and Cox et al. (2013) apply the Duffie and Zhu framework to explore the policy issue of interoperability between CCPs. They examine whether a regulator can reduce expected netted exposures by mandating trades to be novated to a local CCP, which can link to a global CCP that clears a range of products. Both papers retain the assumption of a homogeneous link network, though Cox, Garvin and Kelly allow for some heterogeneity between dealer agents in the magnitude of exposures (but not in the existence of links).

Cont and Kokholm (2014) extend the Duffie and Zhu framework by relaxing the assumption of normal exposures between counterparties and show, using a simulation approach, that Duffie and Zhu's conclusions are sensitive to different distributional assumptions. However, they retain the homogeneous network assumption that Duffie and Zhu use. This is in contrast to our paper, which uses more general and realistic network structures.<sup>4</sup>

<sup>3</sup> See, for example, Sidanius and Žikeš (2012). More recent papers including Duffie et al. (2015) and Campbell (2014) use actual bilateral exposure data to analyze the issue empirically.

<sup>4</sup> While our framework explicitly permits the use of non-normal exposures, some distributional assumption is necessary in order to obtain analytical results.

A recent working paper by [Menkveld \(2015\)](#) considers the Duffie and Zhu framework and looks at the mean and variance of the aggregate exposure of the CCP. His goal is to examine the ability of the CCP to survive simultaneous losses in the centrally cleared asset. We, in contrast, look at the impact of introducing a CCP on the mean and variance of exposures across all asset classes and our focus is on the aggregate exposures of all counterparties across all asset classes, not just those in the centrally cleared asset class.

Our paper makes two key innovations to the Duffie and Zhu model which, to our knowledge, have not been considered before. First, we provide an *analytical generalization* of the model so that it can be applied to any network. Second, we look at how the introduction of a CCP affects the *variance* of counterparty exposures for alternative network structures as well as the mean.

In the broader literature, there are a variety of papers which use a network approach to focus on issues relating to OTC derivatives networks other than netting efficiency. [Markose \(2012\)](#) finds that the empirical OTC derivatives network aggregated over all products can be well-described by a modified core-periphery model and derives summary statistics to identify institutions which carry the greatest quantity of systemic risk. [Heath et al. \(2016\)](#) use the same data set to show that the empirical network structure has the potential to generate stability risks. [Borovkova and Lalaoui El Mouttalibi \(2013\)](#) use a simulation approach to model the effect of the introduction of a CCP on default cascades in a network. They examine both homogeneous and core-periphery networks, and find that homogeneous networks are more resilient.

[Jackson and Manning \(2007\)](#) and [Galbiati and Soramäki \(2012\)](#) use different approaches to examine the desirability of tiering – that is, restricting direct access to the CCP to a limited set of counterparties. [Song et al. \(2014\)](#) extend the Galbiati and Soramäki framework to study the effect of network structure on the maximum exposure risk of the CCP itself and use extreme value theory to obtain analytical results.

Our final results discuss the possibility of agents strategically reforming the network in response to the introduction of a CCP. To our knowledge, there is no existing literature which directly addresses this question. However, recent work by [Aldasoro et al. \(2016\)](#), [Chang and Zhang \(2015\)](#), [Hollifield et al. \(2017\)](#) and [Van der Leij et al. \(2016\)](#) provides some insight into how networks form under strategic interaction.

### 3. A general network model of exposure netting

We assume that the dealer network is not directly observable, but the number of nodes, the degree distribution and the distribution of the magnitude of bilateral exposures is known. This is a realistic assumption for dealer networks, where the regulator and participants often lack exact real-time knowledge of bilateral exposures. This is a generalization of the assumption made in [Duffie and Zhu \(2011\)](#), in which the exact structure of the network is fixed and known.

Let  $N$  be the number of nodes (i.e. market participants) and let  $S$  be a random variable denoting the number of links any given node has.  $S$  takes values on the non-negative integers. Define  $J_i$  to be the set of nodes with which node  $i$  has a link. The size of this set is given by a realization of the random variable  $S$ .

Let  $K$  denote the number of asset classes. Links are undirected: each link is endowed with a vector reflecting the net exposure in each asset class between the two dealer agents. This may be positive or negative, depending on the direction of the exposure. Define  $X_{ij}$  to be a  $K$ -vector of values (weights) on the link between nodes  $i$  and  $j$ , if it exists. The  $k^{\text{th}}$  element in each vector, denoted  $X_{ij}^k$ , represents the net exposure between the two nodes in asset

class  $k$ . When  $X_{ij}^k > 0$  then node  $i$  has a net exposure to node  $j$  in asset class  $k$ , and when  $X_{ij}^k < 0$  then the reverse is true. Each value is generated with the same known distribution, independently of one another and of the link structure of the network.<sup>5</sup>

First consider the situation without a CCP. Consider two linked nodes  $i$  and  $j$ . Define  $Y_{ij}^K \equiv \max\{\sum_{k=1}^K X_{ij}^k, 0\}$  to be the value of node  $i$ 's netted exposure to node  $j$ . Positive net exposures in one asset class can be partially or wholly offset by negative net exposures in another asset class with the same counterparty. If  $i$  and  $j$  are not linked, then the net exposure is zero. The total net exposure of node  $i$  equals  $\sum_{j \in J_i} Y_{ij}^K$ .

Now define the function  $f(K)$  as the expected net exposure between any two linked nodes, given that there are  $K$  asset classes:

$$f(K) \equiv E[Y_{ij}^K]. \quad (1)$$

The expected total netted exposures for a given node  $i$  are:

$$\begin{aligned} \phi_{N,K} &\equiv E\left[\sum_{j \in J_i} Y_{ij}^K\right] = E\left[E\left[\sum_{j \in J_i} Y_{ij}^K \middle| S\right]\right] = E[Sf(K)] \\ &= E[S]f(K), \end{aligned} \quad (2)$$

where we have used the fact that each  $Y_{ij}^K$  is independent from one another and from  $S$ .

Similarly, the variance of the exposure between two nodes after netting is:

$$g(K) \equiv \text{Var}[Y_{ij}^K] \quad (3)$$

and the variance of the total netted exposures of the network is:

$$v_{N,K} \equiv \text{Var}\left[\sum_{j \in J_i} Y_{ij}^K\right]. \quad (4)$$

We can evaluate this expression using the law of total variance:

$$\begin{aligned} v_{N,K} &= E\left[\text{Var}\left[\sum_{j \in J_i} Y_{ij}^K \middle| S\right]\right] + \text{Var}\left[E\left[\sum_{j \in J_i} Y_{ij}^K \middle| S\right]\right] \\ &= E[Sg(K)] + \text{Var}[Sf(K)] = E[S]g(K) + \text{Var}[S]f(K)^2. \end{aligned} \quad (5)$$

#### 3.1. Novating a single asset class to a CCP

Now we introduce a CCP in a single asset class. Without loss of generality, reorder the asset classes so that the centrally cleared asset class is the one labelled  $K$ . The net exposure of a given node  $i$  becomes:

$$\sum_{j \in J_i} Y_{ij}^{K-1} + \underbrace{\max\left\{\sum_{j \in J_i} X_{ij}^K, 0\right\}}_{Y_{i,CCP}^S} \quad (6)$$

where the first term is the sum of a node's exposures to the other nodes and the second term is its netted exposure to the CCP. We can rewrite the second term as  $Y_{i,CCP}^S$ , with the  $S$  superscript arising because the size of  $J_i$  has distribution  $S$ .<sup>6</sup>

Note that, for a given realized value of  $S$ , the two terms in (6) are independent: the first term is determined entirely by exposures arising from the first  $K-1$  assets, while the second is determined entirely by exposures arising from asset  $K$ .

<sup>5</sup> The constraint  $X_{ij}^k = -X_{ji}^k$  does apply, but as our analysis focuses on a representative node, we do not need to apply this constraint.

<sup>6</sup> In [Section 6](#), we consider the same setting but allow the network structure  $S$  to change as a result of the introduction of central clearing.

Now, when there is a CCP, the expected total net exposure of node  $i$  is:

$$\begin{aligned}\widetilde{\phi}_{N,K} &= E \left[ E \left[ \left( \sum_{j \in J_i} Y_{ij}^{K-1} + Y_{i,CCP}^S \right) | S \right] \right] = E[Sf(K-1) + f(S)] \\ &= E[S]f(K-1) + E[f(S)],\end{aligned}\quad (7)$$

and the variance of the total net exposure of node  $i$  is:

$$\begin{aligned}\widetilde{\nu}_{N,K} &= E \left[ \text{Var} \left[ \left( \sum_{j \in J_i} Y_{ij}^{K-1} + Y_{i,CCP}^S \right) | S \right] \right] \\ &\quad + \text{Var} \left[ E \left[ \left( \sum_{j \in J_i} Y_{ij}^{K-1} + Y_{i,CCP}^S \right) | S \right] \right] \\ &= E[Sg(K-1) + g(S)] + \text{Var}[Sf(K-1) + f(S)] \\ &= E[S]g(K-1) + E[g(S)] + \text{Var}[Sf(K-1) + f(S)].\end{aligned}\quad (8)$$

Using (2) and (7), the change in expected net exposure that results from novating a single asset class to a CCP is:

$$\widetilde{\phi}_{N,K} - \phi_{N,K} = E[f(S)] - E[S](f(K) - f(K-1)). \quad (9)$$

Using (5) and (8), the change in variance that results from novating a single asset class to a CCP is:

$$\begin{aligned}\widetilde{\nu}_{N,K} - \nu_{N,K} &= \text{Var}[Sf(K-1) + f(S)] - \text{Var}[S]f(K)^2 \\ &\quad + E[g(S)] - E[S](g(K) - g(K-1)).\end{aligned}\quad (10)$$

These results show that introducing centralized netting will change both the mean and variance of total net exposures. Reduction in either the mean or the variance is likely to be positive for users of the system, since it means that counterparty risk – and total margin needs – are lower either in expectation or volatility. Therefore we can say that:

- centralized netting delivers netting benefits when both (9) and (10) are negative;
- netting is worsened when both expressions are positive; and
- when the expressions have different signs, then there is a trade-off depending on the weight decision-makers place on the mean and the variance of exposures.

The key focus of this paper is to identify the extent to which the effect of centralized netting depends on the underlying network structure.

Note that, in the case where  $K = 1$ , the CCP clears all of the assets that the dealers trade with one another. In this case, we would always expect the introduction of a CCP to improve netting, as it does not disrupt any of the existing bilateral netting sets. This is confirmed by observing that  $\phi_{N,1} \leq \phi_{N,1}$  and  $\widetilde{\nu}_{N,1} \leq \nu_{N,1}$ .<sup>7</sup>

### 3.2. Assigning a distribution to the bilateral exposures

To obtain tractable results for the cases where  $K > 1$ , we need to assume a distribution for the bilateral exposures between dealers. This will enable us to write down expressions for  $f$  and  $g$  in equations (9) and (10). We follow (Duffie and Zhu, 2011) and assume that each of the bilateral exposures  $X_{ij}^k$  is independent and identically distributed normally with mean 0 and variance  $\sigma^2$ .<sup>8</sup> Using

<sup>7</sup> To show this, note that  $f(0) = 0 = g(0)$  and that the  $\max\{\cdot, 0\}$  function is sub-additive.

<sup>8</sup> We can generalize slightly: the results that follow will also apply asymptotically (i.e. as  $N, K \rightarrow \infty$ ) so long as the distribution of the exposures has zero mean and finite variance. This follows from the law of large numbers. In Section 7.1, we show that using Student's  $t$ -distribution instead of the normal distribution does not change the results substantively. This is in line with the findings of Cont and Kokholm (2014).

the formula for the sum of independent normal random variables, we can write the function  $f$  as:

$$f(\theta) = \int_0^\infty \frac{1}{\sqrt{2\pi}\theta\sigma} y e^{-\frac{y^2}{2\theta\sigma^2}} dy = \sigma \sqrt{\frac{\theta}{2\pi}}, \quad (11)$$

and:

$$\begin{aligned}g(\theta) &= \int_0^\infty \frac{1}{\sqrt{2\pi}\theta\sigma} y^2 e^{-\frac{y^2}{2\theta\sigma^2}} dy - \left( \sigma \sqrt{\frac{\theta}{2\pi}} \right)^2 \\ &= \frac{1}{2} \int_{-\infty}^\infty \frac{1}{\sqrt{2\pi}\theta\sigma} y^2 e^{-\frac{y^2}{2\theta\sigma^2}} dy - \sigma^2 \frac{\theta}{2\pi} \\ &= \sigma^2 (\pi - 1) \frac{\theta}{2\pi}.\end{aligned}\quad (12)$$

We substitute these into (9) and (10) to show that a CCP reduces the expectation of net exposures if and only if:

$$\sqrt{K} + \sqrt{K-1} < \frac{E[S]}{E[\sqrt{S}]}, \quad (\star)$$

and introducing a CCP reduces the variance of net exposures if and only if:

$$\text{Var}[S\sqrt{K-1} + \sqrt{S}] < K\text{Var}[S]. \quad (13)$$

Rewriting the first variance term in (13) in terms of expectations operators, we can express this condition as:

$$2\sqrt{K-1} < \frac{\text{Var}[S] - \text{Var}[\sqrt{S}]}{E[S^{\frac{3}{2}}] - E[S]E[\sqrt{S}]}, \quad (\dagger)$$

when the denominator is non-zero.

The denominator of (†) is zero if and only if  $S$  is a constant, which implies that the numerator is also zero. In this case, the degree of every node is constant and known with certainty, so the variance of netted exposures does not change upon introduction of centralized netting. We call such a network a “trivial network”; the Duffie and Zhu network is an example of such a network. Trivial networks are not realistic representations of real-world networks, which typically exhibit some heterogeneity in their degree distributions.

#### 3.2.1. Effect of changing the number of asset classes $K$

**Proposition 1.** *As the number of asset classes  $K$  increases, there is less benefit from the introduction of centralized netting. This is true whether the benefit is measured in terms of lower expectations or lower variance of netted exposures.*

**Proof.** We can express this more precisely as follows: for any number of asset classes  $K$ , let  $\Sigma_K$  be the set of networks for which centralized netting reduces the mean of netted exposures; i.e.  $S \in \Sigma_K$  if and only if  $S$  satisfies (★). Then for all  $K' > K$ ,  $\Sigma_{K'}$  is a subset of  $\Sigma_K$ . The same is true for the variance. The result follows immediately from the fact that the left-hand sides of both expressions (★) and (†) are increasing in  $K$ . □

The intuition behind Proposition 1 is that, as  $K$  increases, the CCP clears a lower proportion of the dealers' activity with one another, so the benefit of netting with the CCP is reduced.<sup>9</sup> This is consistent with the findings of Duffie and Zhu.

For a given network structure, we can define  $K^*$  as the critical value of  $K$  below which expected net exposures decrease upon

<sup>9</sup> See Section 6 for an examination of the robustness of Proposition 1 to a behavioral response to the introduction of a CCP by market participants.



introduction of a CCP, and above which they increase.  $K^\dagger$  is the corresponding critical value for variance. Then:

$$K^* \equiv \frac{1}{4} \left( \frac{E[S]}{E[\sqrt{S}]} + \frac{E[\sqrt{S}]}{E[S]} \right)^2 \quad (14)$$

$$K^\dagger \equiv \frac{1}{4} \left( \frac{\text{Var}[S] - \text{Var}[\sqrt{S}]}{E[S^{\frac{3}{2}}] - E[S]E[\sqrt{S}]} \right)^2 + 1 \quad (15)$$

In other words, expressions  $(\star)$  and  $(\dagger)$  are equivalent to, respectively,  $K < K^*$  and  $K < K^\dagger$ . These expressions are the focus of the analysis moving forward. They relate the impact of introducing a CCP to the number of asset classes  $K$  on the left-hand side, and to the degree distribution  $S$  on the right-hand side.

Next we present a general result on the relationship between  $K^*$  and  $K^\dagger$  that holds for any finite non-trivial network (i.e. a network which has some variance in its degree distribution).

**Proposition 2.**  $K^\dagger > K^*$  for all finite non-trivial networks.

**Proof.** See the Appendix.  $\square$

An implication of Proposition 2 is that the more the decision-maker cares about variance, the wider the range of asset classes  $K$  for which the CCP delivers netting benefits. Relative to the existing literature, which only examines conditions for central clearing to reduce *expected* netted exposures, this proposition suggests that central clearing is more likely to be beneficial, so long as the decision-maker places a non-zero weight on variance.<sup>10</sup>

### 3.2.2. Bounds on $K^*$ and $K^\dagger$

So far in this paper, we have followed the related literature in treating the degree distribution  $S$  as known, even though the exact structure of the network is not. For example, expressions (14) and (15) relate  $K^*$  and  $K^\dagger$  to moments of  $S$ , which are assumed to be known. However, in reality a decision-maker – such as an association of dealers or a regulator trying to decide whether or not to mandate central clearing in a trading network – may lack knowledge even about the degree distribution of the network. For example, a decision-maker may only know the number of asset classes  $K$  and an upper bound on the size of the network  $N$ . In such a case, a decision-maker may be interested in bounds on  $K^*$  and  $K^\dagger$  which can help to inform a decision.

**Proposition 3 (Bounds on  $K^*$  and  $K^\dagger$ ).** Among all networks  $S$  with  $P(S \leq N-1) = 1$ ,  $K^*$  lies between 1 and  $\frac{N^2}{4(N-1)}$ , and both bounds can be achieved. When  $K^\dagger$  is defined, it lies between 1 and  $N-1$ , and both bounds can be achieved.

**Proof.** See the Appendix.  $\square$

A decision-maker can compare the observed  $K$  to the upper bounds, which are both functions of the known quantity  $N$ . If  $K$  is larger than the upper bound on  $K^*$  ( $K^\dagger$ ), then the decision-maker knows that introducing a CCP will increase the mean (variance) of netted exposures. The upper bound for  $K^\dagger$  is at least as high as that for  $K^*$  when  $N \geq 2$ , which means that when the decision-maker is sure that central clearing increases the variance of netted exposures, then she can be sure that it increases the mean too. There may be intermediate cases where a decision-maker is sure that there would be an increase in the mean but cannot be sure about the variance; in such cases, a decision-maker who puts more weight on the effect on variance is more likely to introduce central clearing.

For both  $K^*$  and  $K^\dagger$ , the lower bound is achieved if and only if  $S$  takes values only on 0 and 1, which is a trivial case since the network consists only of disconnected pairs and singletons. An immediate corollary to Proposition 3 characterizes the upper bound on  $K^*$ :

**Corollary 1.** The upper bound on  $K^*$  is achieved if and only if we have a complete, homogeneous network.

As we demonstrate in the next section, this result means that the homogeneous network of Duffie and Zhu (2011) achieves the highest expected netting efficiency under central clearing, compared to all networks of the same size. To put it another way, using the decision criterion in Duffie and Zhu is almost certain to lead to central clearing being accepted too often.

## 4. Examination of different network structures

In this section we consider how the impact of introducing a CCP is affected by the underlying structure of the network. The structure of the network is reflected on the right-hand sides of expressions for  $K^*$  and  $K^\dagger$  via the distribution of the random variable  $S$ .

As a network is a very general object, we restrict our analysis to specific cases identified in the existing literature. First we apply our results to Duffie and Zhu's network and recover their results. Then we apply our analysis to core-periphery and fat-tailed networks, in order to examine the effect on the results of the presence of large well-connected counterparties, which are typically present in real-world financial networks.

### 4.1. Homogeneous network of Duffie and Zhu

Duffie and Zhu (2011) assume the network is completely connected, so every agent has full degree. This means that  $S = N-1$  with certainty, and so  $K^* = \frac{N^2}{4(N-1)}$ , which corresponds to their Eq. (6). The Duffie and Zhu network is a trivial network as defined earlier, and introducing a CCP has no effect on variance. The benefit of the CCP can be considered just by examining the effect on the mean.

A slight generalization of Duffie and Zhu's completely connected network is the Erdős-Rényi random network, where links are formed independently with some probability  $p$ . In this network, the nodes are still homogeneous ex ante but there may be random variations in their realized link patterns. As an Erdős-Rényi network grows in size,  $K^*$  and  $K^\dagger$  grow without limit.<sup>11</sup> However, the Erdős-Rényi homogeneous network is not the focus of our paper, as it is not a good fit to real-world financial networks (as shown, for example, in Markose (2012) and Craig and von Peter (2014)). Instead we focus on two models which have been shown to be more realistic: fat-tailed and core-periphery network structures.

### 4.2. Fat-tailed networks

Many real-world financial networks have degree distributions with significant excess kurtosis. There is likely to be a small number of highly-connected nodes (we can think of these as the major dealers), with the majority of nodes having few connections. One popular model for this property is a 'fat-tailed network': that is, a network with a degree distribution which asymptotically obeys a power law  $P(S = s) \sim s^{-\alpha}$ , for some real-valued parameter  $\alpha > 1$ . Fatter tails are associated with lower values of  $\alpha$ .

In this section we will focus on scale-free networks, which are a particular class of fat-tailed networks that have been shown to

<sup>10</sup> In Section 7.1, we follow (Cont and Kokholm, 2014) in examining the effect of introducing a CCP on Value-at-Risk (VaR), an alternative target statistic.

<sup>11</sup> This is a consequence of Proposition 5 – see Section 4.3.2.

**Table 1**  
Asymptotic behavior of  $K^*$  and  $K^\dagger$  in a fat-tailed network.

	$K^*$	$K^\dagger$
$1 < \alpha \leq 2$	Tends to positive infinity	Tends to positive infinity
$2 < \alpha \leq 3$	Tends to finite limit	Tends to positive infinity
$\alpha > 3$	Tends to finite limit	Tends to finite limit

arise in many real-world applications, including in finance.<sup>12</sup> Focusing on this class is instructive because they arise according to a simple and intuitive process, as explained in the following section.

#### 4.2.1. How do scale-free networks arise?

Barabási and Albert (1999) show that scale-free networks can be formed via growth and preferential attachment. As time goes on, new nodes join the network and tend to form links with the nodes which are better-connected. This is a realistic model of how a derivatives trading network may develop. Over time we would expect new dealers to enter as the market grows. And there are several reasons why these dealers are more likely to trade with the agents which are already better-connected, such as name recognition, an existing relationship in another market, or economies of scale allowing better-connected agents to offer more attractive terms. Barabási and Albert show that networks formed through this process have fat tails with exponent  $\alpha = 3$ .

Barabási and Albert's general solution only applies when the number of nodes becomes very large, but the assumption of a large network is not necessarily realistic for our purposes. For example Duffie and Zhu consider a network of size 12, which is the number of entities that, at the time of writing their paper, had partnered with ICE Trust to create a CCP for clearing credit default swaps.<sup>13</sup> In order to model networks of finite size, we need to settle on a specific form of Barabási-Albert network. We use the scale-free network formation process described in Dorogovtsev et al. (2001); henceforth we refer to this as the DMS network. The major advantage of the DMS network is that it has an exact solution for a network of any size. The DMS network generating process is as follows:

1. Begin at time  $t = 2$  with 3 nodes. Each has two links connecting to one another.
2. Each time period, a new node is added to the network and connects to two existing nodes. To determine which nodes, choose an existing link at random (each with equal probability). The new node then connects to the two nodes which share that existing link. Repeat.
3. This process generates an undirected DMS network. We now need to determine the value of each link. We assume that if two dealers have a link in one asset class, then they have a link in all  $K$  asset classes. For each pair of connected nodes  $i$  and  $j$ , we generate the net exposures  $X_{ij}^k$ ,  $k = 1, \dots, K$ , as  $K$  iid normal random variables with mean 0 and standard deviation  $\sigma$ .

Carried out for  $t$  steps, this produces a network of size  $N = t + 1$ , which tends towards a scale-free network with exponent  $\alpha = 3$  as  $t$  becomes large. The left-hand panel of Fig. 1 shows the realization of a DMS process for  $t = 100$ .

#### 4.2.2. Asymptotic analysis of fat-tailed and scale-free networks

We can use the definition of fat tails to approximate the moments of the degree distribution as the size of the network  $N \rightarrow \infty$ :

$$E[S^m] \sim \int_Z^{N-1} s^{m-\alpha} ds \quad (16)$$

where  $Z$  is some constant.

As  $N \rightarrow \infty$ :

$$E[S^m] \sim \begin{cases} O((N-1)^{m-\alpha+1}) & \text{if } m > \alpha - 1 \\ O(\log(N-1)) & \text{if } m = \alpha - 1 \\ O(1) & \text{if } m < \alpha - 1 \end{cases} \quad (17)$$

and so  $E[S^m]$  has a finite limit if and only if  $m < \alpha - 1$ .

We can apply this result to our expressions for  $K^*$  and  $K^\dagger$ , (14) and (15). Table 1 below summarizes the asymptotics for various values of  $\alpha$ .

When  $\alpha > 3$ , the right-hand sides of both expressions (14) and (15) tend to finite limits. This means that there will be some critical values  $K^*$  and  $K^\dagger$  beyond which the introduction of a CCP will increase the expectation and variance of net exposures no matter how large the network is. But when  $1 < \alpha \leq 2$  the converse is true: for any given number of asset classes  $K$ , there will be some critical network size beyond which the expectation and variance of net exposures decline with the introduction of a CCP. This suggests that a CCP is more likely to deliver benefits to dealers in networks with fatter tails. This makes intuitive sense, because such networks have a large number of major dealers (well-connected nodes), and so the bilateral netting with major dealer is less than efficient in a network where there are fewer of them (i.e. those with thinner tails).

For real-world networks  $\alpha$  typically lies between 2 and 3.<sup>14</sup> This corresponds to the intermediate case in Table 1, implying a potential trade-off between a higher expected value of net exposures and a lower variance when the number of asset classes is larger than the asymptotic limit of  $K^*$ .

**Proposition 4.** For the infinite DMS network, novating a single asset class to a CCP increases expected netted exposures and decreases the variance of netted exposures, for any number of asset classes  $K \geq 2$ .

**Proof.** See the Appendix.  $\square$

The increase in expected exposures that results from the introduction of a CCP ( $\widetilde{\phi}_{N,K} - \phi_{N,K}$ ) has a finite upper bound as  $N \rightarrow \infty$ . However, the reduction in variance ( $\widetilde{v}_{N,K} - v_{N,K}$ ) is without bound. This means that—so long as agents preference functions put any weight on volatility—then, for any given  $K$ , there must be some critical size of the network above which introducing a CCP is beneficial for the dealer agents.

#### 4.2.3. Finite analysis of the scale-free network

The distribution of  $S$  for a given  $t$  is given in Dorogovtsev, Mendes and Samukhin (see their Eq. 8):

$$P_t(s) = \frac{t}{t+1} \left[ \frac{s-1}{2t-3} P_{t-1}(s-1) + \left(1 - \frac{s}{2t-3}\right) P_{t-1}(s) \right] + \frac{1}{t+1} 1_{[s=2]} \quad (18)$$

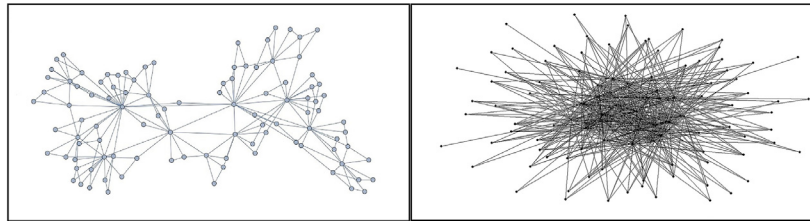
for  $t \geq 3$ , with initial condition  $P_2(s) = 1_{[s=2]}$ . (Here,  $1_{[\cdot]}$  denotes the identity function which takes the value 1 if the condition in the subscript is true, and zero otherwise.)

Fig. 2 shows the effect of introducing a CCP for a range of values of  $K$  and  $t$ . The distribution of  $S$  is determined by Eq. (18) for

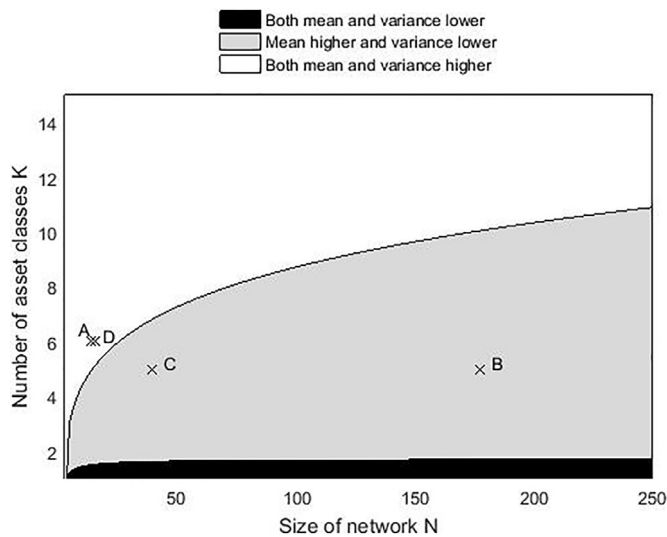
<sup>12</sup> See, for example, (Soramäki et al., 2007; Inaoka et al., 2004) and Garlaschelli et al. (2005).

<sup>13</sup> Sizes of other real-world CCP networks can be found in Table 1 in Galbiati and Soramäki (2012) and footnote 12 in Cox et al. (2013).

<sup>14</sup> For example, Soramäki et al. (2007) and Inaoka et al. (2004) both estimate  $\alpha = 2.1$  for the interbank payment networks in the US and Japan respectively. Garlaschelli et al. (2005) estimate  $\alpha$  lies between 2.2 and 3.0 for networks of shareholdings in the US and Italy. For Barabási-Albert scale-free networks,  $\alpha$  has an asymptotic limit of 3.



**Fig. 1.** Left-hand panel: Scale-free network of size  $t = 100$ , generated using the DMS process. Right-hand panel: Core-periphery network of size  $N = 100$ , generated using a Bernoulli distribution with parameters  $z = 0.25$ ,  $p = .2$ ,  $c_0 = 0$ .



**Fig. 2.** The effect of the introduction of a CCP in a DMS scale-free network, for a range of values of  $(K, t)$ . The chart is calculated up to  $t = 249$  – that is,  $N = 250$ . Values of  $K^*$  are represented by the frontier between the black and gray areas. Values of  $K^\dagger$  are represented by the frontier between the gray and white areas. Illustrative real-world values of  $N, K$  are shown by the cross markers (see footnote <sup>15</sup>).

the corresponding value of  $t$ . This has been calculated for DMS networks with up to 250 members. The upper frontier of the black area in the figure corresponds to  $K^*$ , while the upper frontier of the gray area corresponds to  $K^\dagger$ .

$K^*$  tends monotonically upward with an upper bound of approximately 1.73 (see the proof of Proposition 4 in the Appendix). This confirms that the introduction of a CCP will increase expected net exposures for any non-trivial DMS network (i.e. one where  $K \geq 2$ ). In contrast,  $K^\dagger$  (shown by the frontier of the gray area) increases without limit as  $\sim O((\log(N-1))^2)$ , as predicted by the asymptotic analysis. This means that, for sufficiently large networks, the introduction of a CCP will cause a reduction in the variance of exposures.

The cross markers illustrate values of  $N, K$  for four real-world networks.<sup>15</sup> In some cases the markers are in the white region, meaning that – assuming the underlying network has a DMS structure – the case for introducing a CCP would need to be motivated by considerations other than netting efficiency. In two cases the markets are in the gray region, suggesting that the introduction of a CCP would be beneficial if sufficient weight is placed on the reduced variance of netted exposures.

<sup>15</sup> We have drawn these from published papers where the values of  $N$  and  $K$  are easiest to deduce. These are as follows:– point A ( $N = 12, K = 6$ ): US derivatives network in June 2010 used by Duffie and Zhu (2011); point B ( $N = 176, K = 5$ ): UK interbank network in 2011 used by Langfield et al. (2014); point C ( $N = 38, K = 5$ ): global OTC derivatives network, used by Markose (2012), restricted to most active participants; point D ( $N = 14, K = 6$ ): CDS network in March 2010 estimated from DTCC Trade Information Warehouse Report.

### 4.3. Core-periphery networks

Core-periphery networks have been presented in the recent literature as an alternative to scale-free networks as a model of real-world financial linkages. These networks are characterized by a partition of the nodes into two sets: a heavily-connected set of “core” nodes, along with a sparsely connected set of “peripheral” nodes.

Borgatti and Everett (1999) present a general model to allow for the detection of core-periphery networks: they assume that in such networks all core nodes are linked to one another, while there are no links between peripheral nodes. They then present a statistic to test for correlations between such an idealized core-periphery network and the actual data. Their model is agnostic about the distribution of links between core and periphery nodes; this is because their specification is aimed at empirical verification of the structure, rather than a general model of a core-periphery network.

Langfield et al. (2014) and Craig and von Peter (2014) use the Borgatti–Everett approach to identify core-periphery structures in the UK and German interbank markets respectively. Wetherilt et al. (2010) and Markose (2012) use alternative methods to show that, respectively, the UK money market and the global network of OTC derivatives can be characterized as having core-periphery structures.

#### 4.3.1. How do core-periphery networks arise?

Van der Leij et al. (2016) show that core-periphery structures can arise as the stable outcome of a process of strategic network formation between heterogeneous agents. In their model, there are “big” (core) banks and “small” (peripheral) banks, and the payoff from forming a link with a big bank is greater than a link with a small bank. Chang and Zhang (2015) demonstrate that, when agents differ in the amount that they need to trade, they will self-organise into a core-periphery structure, with the agents with the lowest trading need forming the core. Chang and Zhang associate these core nodes with market makers.

Underlying this network generation process is the assumption that, for any given agent, links to core nodes are desirable, while links to peripheral nodes are not. There are plausible reasons why this may be the case for real-world financial networks. Agents may prefer to deal with larger players, with whom they are likely to have existing relationships in other markets. Exposures to larger players may be easier to monitor. And economies of scale may mean that these larger players offer more attractive trading terms.

Abstracting away from consideration of individual nodes’ optimal strategies, we can characterize the formation of a core-periphery network using the following simple process:

1. Begin with  $c_0$  core nodes, which are connected to each other.
2. At each step a new node is added. With probability  $z$  this new node is labeled ‘core’. Otherwise, the node is labeled ‘peripheral’.

3. A new core node forms links with all of the existing core nodes with certainty, and forms links with each of the existing peripheral nodes according to some distribution  $W$ . A new peripheral node will never form links with existing peripheral nodes, but will form links with existing core nodes according to the distribution  $W$ .

This process, carried out for  $N - c_0$  steps, will produce a network of size  $N$  which meets the Borgatti-Everett definition of a core-periphery network. The parameters of the model are  $c_0$ ,  $z$  and the distribution  $W$ . Borgatti and Everett allow any feasible distribution to determine core-periphery links; for example  $W$  could depend on existing links in the network at a given point in time. One natural and simple way to model links between the core and periphery is to assume that each link occurs independently with some fixed probability  $p \in (0, 1)$  – that is, the link formation process follows a Bernoulli distribution. Under this assumption, the number of links for any randomly chosen node can be expressed as a mix of binomial distributions plus a constant.<sup>16</sup> The right-hand panel of Fig. 1 shows a realization of such a network-generating process. We will focus on the Bernoulli core-periphery network in the remainder of this section.

#### 4.3.2. Asymptotic analysis of core-periphery networks

In order to make asymptotic inferences about the Bernoulli core-periphery network, we will state and prove a more general Proposition on the asymptotic limit of networks with ‘thin tails’, which we define as follows.

**Proposition 5.** Suppose the degree distribution  $S$  of the network has the following properties:

- $\text{Var}[S]/E[S]^2$  tends to a finite limit as  $N \rightarrow \infty$ ;
- All of the higher moments tend to zero as  $N \rightarrow \infty$ .

Then  $K^*$  and  $K^\dagger$  are both  $\sim O(E[S])$  as  $N \rightarrow \infty$ .

**Proof.** See the Appendix.  $\square$

For a network which meets the conditions in Proposition 5,  $K^*$  and  $K^\dagger$  will increase without bound as the size of the network becomes large. This suggests that, for any given number of asset classes  $K$ , there is a minimum size of the network above which the introduction of a CCP would reduce both the mean and variance of net exposures.

Examples of network models which meet these “thin-tailed” conditions are the Duffie and Zhu network, the Erdős-Rényi network described in Section 4.1, and the Bernoulli core-periphery network described above. In all of these cases,  $E[S] \sim O(N)$ .

Proposition 5 does not apply to a network with a link formation process  $W$  which generates fat tails; such a network would not meet the conditions stated in the Proposition. For example, if the core and peripheral nodes form links according to a Barabási-Albert preferential attachment process, then such a network will exhibit asymptotic results similar to those derived in Section 4.2.2. Markose (2012) shows that the network of global OTC derivatives exposures can be modeled by a core-periphery network with fat tails in the degree distribution. In this case, the analytics of the fat-tailed network would be more appropriate.

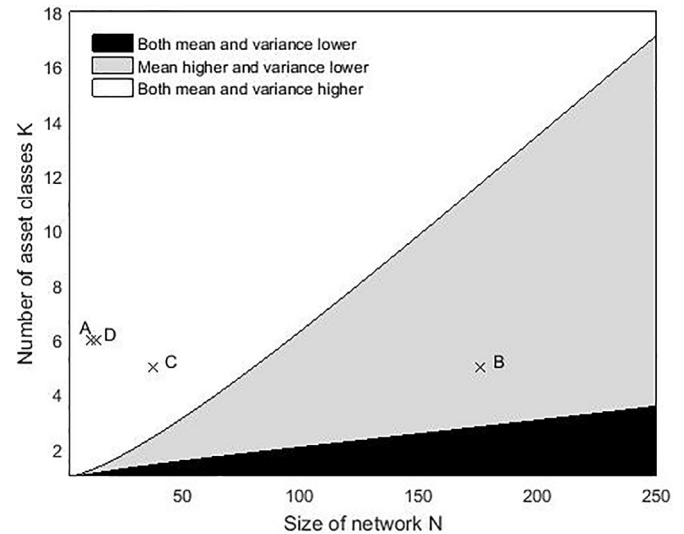
#### 4.3.3. Finite analysis of the Bernoulli core-periphery network

While the only parameter in the DMS network is its size  $N$ , the Bernoulli core-periphery network has four parameters  $N$ ,  $z$ ,  $p$  and

**Table 2**

Parameter estimates for selected real-world core-periphery networks.

	Global OTC	Dutch interbank	German interbank
Period	2009 Q4	2008 Q4	2003 Q2
$N$	204	100	1802
Estimated $z$	0.10	0.15	0.02
Estimated $p$	0.01	0.31	0.11



**Fig. 3.** The effect of the introduction of a CCP in a core-periphery network with  $z = 0.10$ ,  $p = .10$ ,  $c_0 = 0$ , for a range of values of  $(K, N)$  up to  $N = 250$ . Values of  $K^*$  are represented by the frontier between the black and gray areas. Values of  $K^\dagger$  are represented by the frontier between the gray and white areas. Illustrative real-world values of  $N, K$  are shown by the cross markers (see footnote <sup>15</sup>).

$c_0$ . In order to derive numerical solutions, we need to choose feasible parameter values, so we turn to the empirical literature. Table 2 below summarizes parameter estimates from three selected papers: the global OTC derivatives network from Markose (2012), the Dutch interbank market from Van der Leij et al. (2016), and the German interbank market from Craig and von Peter (2014).<sup>17</sup> The value of  $c_0$  is impossible to observe and is likely to make little difference for larger networks, so we assume  $c_0 = 0$ . We will use  $z = 0.10$ ,  $p = .10$  for our numerical solutions – these parameter values are fairly representative of Table 2.<sup>18</sup>

Fig. 3 shows the effect of introducing a CCP for various values of  $N$  up to 250, for the parameters  $z = 0.10$ ,  $p = .10$ ,  $c_0 = 0$ . As predicted,  $K^*$  and  $K^\dagger$  increase approximately linearly with  $N$ . There is a gray intermediate area where  $K^* < K < K^\dagger$  – in such cases, introducing a CCP will increase expected net exposures in the network, but reduce the variance. The cross markers represent the same four real-world networks shown on Fig. 2.<sup>19</sup>

Comparing Figs. 2 and 3, we can see that network structure has an important effect on the benefits of introducing a CCP. Compare

<sup>17</sup> For the global OTC network we use the “inner core” as defined by Markose. Each paper provides the number of nodes  $N$ , the size of the core  $C$  and the total number of directed links  $L$ . We assume that  $c_0 = 0$ . We can then estimate  $z = \frac{C}{N}$  and  $p = \frac{L - C(C-1)}{2C(N-C)}$ .

<sup>18</sup> The Markose parameter values may be thought to be the best proxy, given that we are interested in central clearing of OTC derivatives and not other forms of interbank lending. However, it should be noted that the Markose network exhibits fat tails and so does not conform very well to a Bernoulli core-periphery structure.

<sup>19</sup> We use different networks in Table 2 to those used to illustrate the figures. This is due to differing degrees of confidence in our ability to estimate the relevant parameters, given the data and information provided in the original papers. The networks in Figs. 2 and 3 are those for which we can most confidently estimate  $N$  and  $K$ . The networks in Table 2 are those for which we can most confidently estimate  $N, z$  and  $p$ .

<sup>16</sup> Let  $C$  be the number of core nodes. Then  $C \sim c_0 + \text{Bin}(N - c_0, z)$ , and the conditional distribution  $S|C \sim \frac{C}{N} \cdot [(C-1) + \text{Bin}(N-C, p)] + (1 - \frac{C}{N}) \cdot \text{Bin}(C, p)$ . Note that the case  $z = 1$  corresponds to the Duffie and Zhu network with size equal to  $N$ .



point C in the two networks. This represents the global OTC derivative network explored in Markose (2012). This point lies in the gray region in Fig. 2 and the white region in Fig. 3. If we assume a DMS network structure, then introducing a CCP to this network increases the mean of netted exposures, but reduces their variance. Therefore a sufficiently risk-averse policymaker may be willing to introduce a CCP. But, if we assume a core-periphery structure, then introducing a CCP increases both the mean and variance, so even a highly risk-averse policymaker would not wish to introduce a CCP. Clearly, the network structure ought to be an important factor in policymakers' decisions about whether to mandate central clearing.

As Fig. 3 shows, there are network sizes  $N$  where a CCP is unambiguously bad for netting efficiency. For all  $N \leq 93$ ,  $K^* < 2$  and so a CCP will never reduce average net exposures for networks of this size. And for all  $N \leq 29$ ,  $K^\dagger < 2$  and so introducing a CCP for networks of this size increases both the mean and variance of net exposures.

Testing other parameter values, we also find that  $K^*$  and  $K^\dagger$  always increase with  $N$  and with both  $z$  and  $p$ . We can interpret higher  $K^*$  and  $K^\dagger$  as indicating that the introduction of a CCP brings greater netting benefits. These results arise because, when the network relies on links with a small fraction of key nodes (the scale-free case), bilateral netting substantively reduces net exposures, and central clearing disrupts this. But when links are more spread amongst a larger number of key nodes (the core-periphery case with large  $z$  or  $p$ ), central clearing may bring greater netting benefits.<sup>20</sup>

This strongly suggests that knowledge of network structure – as well as the size and the number of asset classes – is important in assessing the possible benefits of central clearing on netting efficiency. Particularly important is the tail of the degree distribution, which determines the connectivity of the largest nodes in the network.

## 5. Generalization to several CCPs and multiple asset classes

Until now we have only considered the case of the introduction of centralized netting in one asset class. In reality, a single CCP may clear more than one asset class, or there may be several CCPs all clearing different asset classes. This may affect netting efficiency: for example Duffie and Zhu (2011) show that merging multiple CCPs into a single CCP will always improve netting efficiency. In this section, we briefly show how our results can be generalized to a setting with several CCPs.

Suppose that, as before, there are  $N$  dealer agents in the OTC derivatives network, and  $K$  asset classes.  $M$  CCPs are introduced, indexed by  $m = 1, \dots, M$ . CCP  $m$  clears a number of asset classes  $a_m$ . We relabel the asset classes so that asset classes in the set  $\{1, \dots, a_1\}$  are cleared by CCP 1, those in the set  $\{a_1 + 1, \dots, a_1 + a_2\}$  are cleared by CCP 2, and so forth.

The total number of asset classes novated to central clearing is  $A := a_1 + \dots + a_M$ . The asset classes with labels  $\in \{A + 1, \dots, K\}$  are not centrally cleared.<sup>21</sup>

By the same reasoning as in Section 3.1, we can show that the expectation and variance of exposures following centralized net-

ting are:

$$\widetilde{\phi}_{N,K} = E[S]f(K - A) + \sum_{m=1}^M E[f(a_m S)], \quad (19)$$

$$\begin{aligned} \widetilde{v}_{N,K} = E[S]g(K - A) + \sum_{m=1}^M E[g(a_m S)] \\ + \text{Var}\left[Sf(K - A) + \sum_{m=1}^M E[f(a_m S)]\right]. \end{aligned} \quad (20)$$

Note that in the case  $m = 1$ ,  $a_1 = 1$ ,  $A = 1$ , we recover Eqs. (7) and (8).

These equations provide us with a way of comparing the netting efficiency under different constellations of CCPs, by writing a given constellation in terms of a partition of the set of asset classes. Let us assume again that the underlying bilateral exposures  $X_{ij}^k$  are iid normally distributed  $\sim N(0, \sigma^2)$  for some parameter  $\sigma$ . We can use the expressions (11) and (12) to show that introducing a given CCP constellation reduces the expectation and variance of exposures if and only if, respectively:

$$\frac{1}{A} \left( \sqrt{K} + \sqrt{K - A} \right) \sum_{m=1}^M \sqrt{a_m} < \frac{E[S]}{E[\sqrt{S}]}, \quad (\star')$$

$$2\sqrt{K - A} < \frac{A \text{Var}[S] - \left( \sum_{m=1}^M \sqrt{a_m} \right)^2 \text{Var}[\sqrt{S}]}{\left( \sum_{m=1}^M \sqrt{a_m} \right) (E[S^{\frac{3}{2}}] - E[S]E[\sqrt{S}])}. \quad (\dagger')$$

Expressions  $(\star')$  and  $(\dagger')$  are generalizations of  $(\star)$  and  $(\dagger)$ , respectively. As before, all of the  $K$  terms are on the left-hand sides of the expressions, which are increasing in  $K$ . Therefore a given CCP constellation will tend to be less efficient when the total number of asset classes is higher. The intuition is the same as before: when  $A$  is fixed and  $K$  increases, then the CCPs collectively clear a smaller proportion of the network, and so the benefits they bring from netting within asset classes are less likely to be greater than the cost from disruption of existing bilateral netting sets across asset classes. We can redefine  $K^*$  and  $K^\dagger$  as the values of  $K$  for which there is equality between the left- and right-hand sides of  $(\star')$  and  $(\dagger')$ , respectively.

We can use these expressions to prove the following general results.

### Proposition 6.

1. Merging CCPs will always improve netting efficiency: Given any constellation  $\Psi_1 = \{a_1, \dots, a_M\}$ , consider a new constellation  $\Psi_2 = \{a_1 + a_2, \dots, a_M\}$  with one fewer CCP. Then both the expectation and variance of exposures under  $\Psi_2$  will be lower than under  $\Psi_1$ .
2. It is most efficient to novate asset classes to the largest existing CCP: Given any constellation  $\Psi_1 = \{a_1, \dots, a_M\}$ , consider two alternative constellations  $\Psi_2 = \{a_1 + 1, a_2, \dots, a_M\}$  and  $\Psi_3 = \{a_1, a_2 + 1, \dots, a_M\}$ . If  $a_1 > a_2$ , then both the expectation and variance of exposures under  $\Psi_2$  will be lower than under  $\Psi_3$ .

**Proof.** See the Appendix.  $\square$

Proposition 6 shows that merging CCPs will reduce both the mean and variance of net exposures, and that it is most efficient to clear a new asset class through the largest existing CCP. Taken together, these findings imply that the most efficient arrangement would be to have a single CCP net every asset class. It is easy to see that this is best, because all exposures can be netted against one another. However, such an arrangement may not be optimal once a regulator takes into account systemic risk because such a CCP would be hugely systemically important. If a regulator does

<sup>20</sup> This is supported by Table 1 which suggests that, as  $\alpha$  increases in a large fat-tailed network, central clearing tends to bring greater netting benefits. That is because a fatter tail means a greater number of nodes with a substantial number of links, so bilateral netting with individual nodes is relatively less important.

<sup>21</sup> We assume here that any given asset class is cleared by at most one CCP. We do not consider the case where an asset class can be cleared by more than one CCP. Our results in this section suggest that such a case is inefficient.

not want too many asset classes being cleared by a single CCP, our expressions ( $\star'$ ) and ( $\dagger'$ ) can help her to compare different potential clearing arrangements and to assess trade-offs against some other factor of concern, such as systemic risk.

## 6. Strategic behavior and asset dynamics

Until now we have assumed that the structure of the network is not affected by the introduction of central clearing, except for the trades that are novated. Nor does the distribution of the exposures change. In reality, dealer agents may respond to the introduction of a CCP by adjusting their links with other agents, and changing their bilateral exposures. In this section we allow the introduction of central clearing to change the network structure and the exposures.<sup>22</sup>

We make a distinction between the *trading network* and the *clearing network*. The trading network reflects the structure of derivative trading relationships prior to the introduction of centralized netting. The clearing network reflects the structure after centralized netting. So far, we have assumed that the two networks are identical. Suppose instead that each network reflects the equilibrium result of a strategic network formation process. The dealer agents trade off various effects, such as:

- Bilateral netting benefits: to maximize bilateral netting and thus economize on margin costs, each dealer agent prefers to trade more heavily with each counterparty, and thus have relatively few links.
- Search costs: to minimize search costs, each dealer agent prefers to have fewer links, and consequently to trade more with each counterparty. Links to core nodes entail lower search costs.
- Counterparty risk management: to minimize counterparty credit risk, each dealer agent seeks to diversify its counterparties, forming more links and trading less with each.

As a result of these trade offs, agents form links with several counterparties, but do not distribute their exposures evenly. This gives rise to a particular trading network structure. In fact, we may suppose that the network structures that were studied in the previous sections were the result of optimizing behavior. Now let us suppose, that following the introduction of central clearing, the agents are given the opportunity to re-form the network.

To our knowledge, there is no existing academic literature – either theoretical or empirical – addressing the question of how the clearing and trading networks might differ following the introduction of central clearing. As discussed above, it is likely to depend on agents preferences and so will be an empirical question. Our approach is to remain agnostic about the answer to this question, and instead consider how our model can be adapted to a given clearing network.

### 6.1. Effect on expected net exposures post-CCP

Let  $S$  denote the degree distribution in the trading network, and  $\tilde{S}$  the degree distribution in the clearing network. Similarly, let  $\sigma^2$  and  $\tilde{\sigma}^2$  denote the variance of each bilateral exposure in the trading and clearing network, respectively. Let  $f(\theta)$  denote the expected net exposure between any two linked nodes in the trading network, given that there are  $\theta$  asset classes traded between them. Let  $\tilde{f}(\theta)$  denote the expected net exposure in the clearing network. This formulation allows not only the structure of the network to change, but also the volatility of the in-the-money positions of the

contracts, which may reflect, for example, changes in the magnitudes of bilateral net positions dealer agents are willing to take against one another.<sup>23</sup>

In the trading network, the expected total netted exposures for a given node  $i$  are:

$$\phi_{N,K} = E[S]f(K) = \sigma \sqrt{\frac{K}{2\pi}} E[S]. \quad (21)$$

In the clearing network, the corresponding quantity is:

$$\begin{aligned} \tilde{\phi}_{N,K} &= E[\tilde{S}]\tilde{f}(K-1) + E[\tilde{f}(\tilde{S})], \\ &= \tilde{\sigma} \sqrt{\frac{K-1}{2\pi}} E[\tilde{S}] + \tilde{\sigma} \sqrt{\frac{1}{2\pi}} E[\sqrt{\tilde{S}}]. \end{aligned} \quad (22)$$

Define  $\Delta$  to be the change in expected net exposures after the introduction of central clearing that we defined in (9); i.e.:

$$\begin{aligned} \Delta &= \tilde{\phi}_{N,K} - \phi_{N,K}, \\ &= \frac{1}{\sqrt{2\pi}} \left( \tilde{\sigma} \left( \sqrt{K-1} E[\tilde{S}] + E[\sqrt{\tilde{S}}] \right) - \sigma \sqrt{K} E[S] \right). \end{aligned} \quad (23)$$

Central clearing reduces expected net exposures if and only if  $\Delta < 0$ . When  $S = \tilde{S}$  and  $\sigma = \tilde{\sigma}$ , this is the same condition ( $\star$ ) as in the case when the clearing and trading networks were identical.

**Proposition 7.** Let  $\tilde{\sigma} E[\tilde{S}] = \beta \sigma E[S]$ .  $\Delta$  is increasing with respect to  $K$  if and only if  $\beta > (\frac{K-1}{K})^{\frac{1}{2}}$ .

**Proof.** Substitution of  $\beta \sigma E[S]$  into (23) and partially differentiating with respect to  $K$  yields the expression  $\frac{\sigma E[S]}{2\sqrt{2\pi}} [\beta(K-1)^{-\frac{1}{2}} - K^{-\frac{1}{2}}] > 0$ .  $\square$

Proposition 7 is a generalization of the result in Proposition 1, which states that the introduction of centralized netting delivers fewer benefits as the number of asset classes increases. This result is robust to endogenous changes in the trading network so long as the expected number of linkages and the variance of exposures in clearing network are not too much smaller than in the trading network (so that  $\beta$  satisfies the condition given in the statement of Proposition 7).

Interestingly, the possibility that traders will reduce the number of linkages or the variance of exposures in response to central clearing opens up the possibility that central clearing may be better when  $K$  is high. When the trading and clearing networks are the same, then lower  $K$  makes central clearing more likely to be efficient, because the CCP clears a larger proportion of derivatives. But this may no longer apply if the network changes. Suppose that the clearing network is sufficiently less connected than the trading network, so that the condition in Proposition 7 is violated. Then, even without considering the impact of the CCP itself, the clearing network has greater bilateral netting efficiency than the trading network because each agent has her trades concentrated among fewer counterparties. When the CCP is introduced, these bilateral netting sets are disrupted. The higher  $K$  is, the less disruption is caused by the introduction of central clearing in one asset class, meaning a more efficient network.

To illustrate this point, suppose that we have a simple two-agent trading network with degree distribution  $S \sim \text{Bin}(n, p)$  and that the clearing network has degree distribution  $\tilde{S} \sim \text{Bin}(n, \tilde{p})$ . Let  $\sigma = \sqrt{2\pi}$  and let  $\tilde{\sigma} = \tau \sqrt{2\pi}$ . Then:

$$\Delta = \tau n \tilde{p} \sqrt{K-1} - np \sqrt{K} + \tau E[\sqrt{\tilde{S}}] \quad (24)$$

<sup>22</sup> Sections 6 and 7 were added to the paper at the encouragement of the editor and referee.

<sup>23</sup> We continue to assume exposures are determined by the same process as in Section 3.2. However, here we are allowing agents to re-optimize in a way that would change the variance of these exposures.

which is increasing in  $K$  when:

$$\frac{1}{K} > 1 - \left( \frac{\tilde{p}\tau}{p} \right)^2. \quad (25)$$

Consider the case  $\tilde{p}\tau < p$ . Then there is some  $\bar{K}$  such that condition (25) fails for all  $K > \bar{K}$  and so central clearing is more efficient when there are more asset classes in the market. This occurs either because the clearing network is less connected than the trading network, or it has smaller bilateral exposures (or both). Either way, it means that bilateral netting in the clearing network is more efficient than in the trading network. Introducing a CCP in one asset class disrupts that. But if the one asset class is only a small proportion of the total market, then the impact is less egregious.

We conclude that it is important for policymakers considering the introduction of CCPs to be aware not only of the trading network structure, but also to try to anticipate how dealer agents' bilateral relationships may change following the introduction of a CCP. For example, suppose that a policymaker believes that a likely consequence of introducing a CCP is that agents increase their bilateral risk positions vis-à-vis one another. Then the clearing network will be less connected than the trading network and have more volatile bilateral exposures across the links that do exist. There is then a risk that netting efficiency may be worse as a consequence of the introduction of central clearing, not better.

The same analysis can be applied to our results in Section 5 too. In the standard model without strategic network formation, centrally clearing more asset classes is always better than clearing fewer. However, that may no longer be the case if each additional asset class further concentrates the clearing network.

In this section we have focused on the effect of strategic network formation on expected net exposures, but a similar intuition can be applied when considering the variance, or indeed any other feasible summary statistic.

## 7. A computational approach to assessing the benefits of central clearing

In this section, we show how our model can be used by a practitioner – for example, a policy maker who wishes to determine whether or not the introduction of central clearing will improve netting efficiency. We present a computational approach which allows for the estimation of the benefits with knowledge of only a few parameters, rather than the entire network structure. We allow the trading and clearing networks to be different, as described in Section 6. We focus here on measuring the change in expected netting efficiency,  $\phi_{N,K} - \phi_{N,K}$ , but it is straightforward to extend the framework to measuring changes in the variance of netted exposures too, or indeed any other feasible summary statistic.

We make an assumption that the overall willingness of each agent to bear market risk does not change following the introduction of central clearing. Market risk is defined as the volatility of an agent's total exposures in each asset class. This measures an agent's need or desire to trade each asset class. This is based on a premise that participation in the market is not affected by the introduction of the CCP, although the choices of counterparties may be.

This assumption means that each agent bears the same expected market risk in the system in both the trading and clearing network. This is in contrast to *counterparty credit risk*, which a CCP can and generally does affect.

**Proposition 8.** *If agents bear the same expected market risk before and after the introduction of central clearing, then  $\sigma^2 E[S^2] = \tilde{\sigma}^2 E[\tilde{S}^2]$ .*

**Proof.** See the Appendix. The idea of the proof is as follows. For each agent and each asset class, market risk is normally distributed

with mean zero and variance  $\sigma^2 S^2$ , conditional on  $S$ . The unconditional variance of the market risk is – by assumption – the same in the trading and clearing network, so the desired equality is obtained.  $\square$

Proposition 8 alone does not allow us to pin down the degree distribution of the clearing network, because it only tells us how the second moment will change. We can make an additional assumption that both the trading and clearing networks have a core-periphery structure, as described in Section 4.3. There is plenty of empirical and theoretical support for this assumption. The empirical literature has already been discussed in Section 4.3. A number of recent theoretical papers have shown that core-periphery structures can arise in equilibrium as a result of heterogeneity between agents.<sup>24</sup> In these papers, agents differ in terms of liquidity needs or urgency of trade. Those agents who have less need to trade take on the role of a market maker and form the core. The agents who have greater need to trade form the periphery.

**Assumption 1.** The relationship between the trading and clearing networks is governed by the following constraints:

1. The trading network is a core-periphery network with parameter set  $(N, z, p, \sigma)$ . The clearing network is a core-periphery network with parameter set  $(\tilde{N}, \tilde{z}, \tilde{p}, \tilde{\sigma})$ . For simplicity, we make the initial number of core nodes  $c_0$  equal to 0.
2. The number of dealer agents is the same in both networks:  $N = \tilde{N}$ .
3. The proportion of core nodes is the same in both networks:  $z = \tilde{z}$ .
4. Market risk is the same in both networks; that is, Proposition 8 holds.

The number of dealer agents can be assumed to be exogenously determined and not affected by the introduction of central clearing. Similarly, the core nodes are assumed to have intrinsic differences from the peripheral nodes, and so they make up the same proportion of both networks. For example, the core nodes may represent major broker-dealers who have relationships with a large number of smaller financial institutions and non-financial customers in a range of different markets. These existing relationships mean lower search costs for their counterparties, and so they tend to form links with each other whether or not a CCP is introduced.

Given the parameter set of the trading network, the clearing network has four parameters and three constraints. This makes the set of feasible clearing networks a one-dimensional set. We use  $\tilde{p}$  to parameterize this set. Clearing networks with higher values of  $\tilde{p}$  have more connections. By Proposition 8, that implies that each bilateral link will have a lower variance.

Conditional on a number of core nodes  $C \sim \text{Bin}(N, z)$ , the trading network has the following distribution:

$$S|C \sim \frac{C}{N}(C-1 + \text{Bin}(N-C, p)) + \left(1 - \frac{C}{N}\right)\text{Bin}(C, p), \quad (26)$$

and similarly for the clearing network. It is straightforward to compute the unconditional mean of  $S$ :

$$E[S] = (N-1)z(z + 2p(1-z)), \quad (27)$$

but the other unconditional moments are more complicated. To assess the conditions for a reduction of net exposures we need half-moments, which do not exist in closed form. Therefore it will be necessary to take a computational approach.

<sup>24</sup> See, for example, Aldasoro et al., 2016; Chang and Zhang, 2015; Hollifield et al., 2017 and Van der Leij et al. (2016).

**Table 3**

Robustness checks using a core-periphery network with parameters  $N = 38, K = 5, z = 0.10, p = .10, c_0 = 0$ . The table shows the mean, variance and 99% VaR of netted exposures before and after the introduction of a CCP, under various assumptions. These results are obtained computationally by simulating the network one million times in each case.

		Base model	Correlated	Different variance	Student's $t$
Mean	No CCP	0.62	0.74	2.29	0.56
	With CCP	0.75	0.84	0.94	0.65
	Relative change	0.20	0.14	-0.59	0.17
Variance	No CCP	2.58	4.71	35.11	2.40
	With CCP	2.91	4.96	4.37	2.64
	Relative change	0.13	0.05	-0.88	0.10
99% VaR	No CCP	7.79	10.84	28.65	7.40
	With CCP	8.17	11.11	9.92	7.70
	Relative change	0.05	0.02	-0.65	0.04

To compute the netting efficiency pre- and post-CCP, a planner needs estimates of the parameters ( $N, z, p, \sigma$ ), as well as the number of asset classes  $K$ . We suggest that these can all be estimated using summary statistics, and thus do not require knowledge of the full network. Post-crisis, regulators are obtaining better information about trading networks, and so estimation of these parameters should be possible. For further details, see empirical papers such as [Craig and von Peter \(2014\)](#). The results of the previous section suggest that outcomes may be non-monotonic in the parameters, so having precise estimates for the parameters is very important.

As  $\tilde{p}$  is only observable ex post, a planner's best approach is to form a prior over the likely connectivity of the clearing network and use that to compute a prior distribution for netted exposures. Future empirical research may help to refine priors over  $\tilde{p}$ , as regulators can look at the effect of introducing central clearing in other networks.

### 7.1. Robustness checks

We use our computational approach to check whether any of our modelling assumptions drive the results.<sup>25</sup> In particular, we follow ([Cont and Kokholm, 2014](#)) in looking at how the results may change under the following circumstances:

1. exposures are positively correlated;
2. exposures in different asset classes are not identically distributed;
3. a distribution other than the normal is used to generate exposures;
4. the effectiveness of central clearing is assessed by comparing Value-at-Risk (VaR) of netted exposures, rather than the mean or variance.

We examine these separately, using the global OTC derivatives network (identified as point C in [Figs. 2 and 3](#)) and assuming a core-periphery structure with  $N = 38, K = 5, z = 0.10, p = .10, c_0 = 0$ . We follow ([Cont and Kokholm, 2014](#)) in implementing each of the robustness checks. For the first, we assume that exposures are normally and identically distributed with unit variance but with correlation coefficient  $\rho = .1$ . For the second, we assume that there is zero correlation but exposures in the centrally cleared asset class  $K$  has double the standard deviation of exposures in the other asset classes. For the third, we use the Student's  $t$ -distribution with 3 degrees of freedom and unit variance.<sup>26</sup> For the fourth, we use

99% VaR to report all of the previous results. For each of these, we generate the network one million times and report the statistics in [Table 3](#).

As shown by point C in [Fig. 3](#), both mean and variance increase upon introduction of a CCP. Using VaR leads to similar decisions as using the other two metrics. For these parameters, assuming positively correlated exposures or using Student's  $t$  makes the CCP mildly more effective (i.e. a smaller increase in the metric), but the direction of the change is the same.

The most striking case is when exposures in asset class  $K$  has a higher variance than the other asset classes. Under these parameters, introducing a CCP leads to a substantive improvement in netting under all metrics, meaning that a different decision would be made. The reason is that exposures in asset class  $K$  are larger than those in other asset classes, so netting them against each other (in a CCP) is more efficient than netting them against smaller exposures (as with bilateral netting). If, instead, exposures in asset class  $K$  had a lower variance, introducing a CCP would be less effective. All of these results are in line with the findings of [Cont and Kokholm \(2014\)](#).

## 8. Implications for policy

In 2009, G-20 Leaders called for central clearing in a variety of derivatives markets (in particular high-volume standardized credit default and interest rate swaps). This has now been introduced into legislation in various member countries; for example in the United States through Title VII of the Dodd-Frank Act of 2010 and in the European Union through the European Market Infrastructure Regulation (EMIR) of 2012. What can our analysis tell us about the need for regulatory intervention? In other words, given that the dealer agents have chosen not to set up a CCP themselves, what market failure does a regulator address by mandating central clearing?

Margin requirements in a derivatives network are related to netted exposures. Let us assume that the dealers are less averse to volatility and the associated extreme outcomes than is socially optimal. This may be the case, for example, if high or volatile margin requirements impose externalities on markets for the collateral assets ([Murphy et al., 2014](#)), or if agency problems mean that dealers take excessive risks. In such cases, the dealers would not wish to introduce centralized netting, even though it may be socially optimal to do so. A social planner may determine that the optimal policy measure is to mandate central clearing, although the dealers themselves may disagree. This explains the need to introduce regulation which mandates that dealers use a CCP.

<sup>25</sup> We thank an anonymous referee for this suggestion.

<sup>26</sup> See [Cont and Kokholm \(2014\)](#) for an explanation of this assumption. Using a distribution with infinite variance or mean will make the results heavily dependent on a few rare extreme observations.



Following the United Kingdom's vote in June 2016 to leave the European Union, the European Commission is considering changes to the oversight of the clearing of euro-denominated swaps, most of which is done in London.<sup>27</sup> One likely option is a mandate that clearing of such swaps must be done within the European Union. This would mean disruption to existing netting sets among CCPs in the UK and EU (see O'Malia (2017)). Our model can help to assess the possible impact of this change on netting efficiency in these CCPs. Proposition 6 predicts that the outcome of this will be to make London-based CCPs less efficient and the beneficiaries of this in the EU more efficient. However, as London is currently the larger financial centre, the overall effect will be worse aggregate netting efficiency.

## 9. Concluding remarks

We show how the introduction of centralized netting in a single asset class affects both the mean and variance of netted dealer exposures, depending on the underlying structure of the network. Centralized netting is more likely to decrease both mean and variance if the network is larger, or if there is a smaller number of asset classes traded in the network. However, centralized netting brings fewer netting benefits if the network relies on a small number of key nodes for most of its links.

This has welfare implications because net dealer exposures relate to aggregate counterparty risk in the network and to liquidity needs. For example, if centralized netting reduces both the expectation and variance of net exposures, it is likely to be beneficial. If centralized netting increases both the expectation and variance of net exposures, then it is likely to reduce welfare. We show that there will be cases where the decision-makers must trade off lower variance against a higher mean, but not vice versa. This means that some aversion to volatility may be required to favor the introduction of a centralized netting system, such as a CCP. In such cases, whether centralized netting improves welfare or not will depend upon details of risk controls, as well as the preferences of the dealers, policymakers and any other relevant agents.

A criticism of work that analyzes changes in financial network structures that result from shocks or policy changes is that nodes in the network (financial institutions) are not assumed to optimally adjust their behavior following the event studied. This practice arises in part because of difficulties associated with accurately modeling optimal network formation in particular market scenarios. We show that there are trade-offs that can lead to ambiguous conclusions. Our approach is to remain agnostic about the outcome of behavioral effects, but to identify criteria under which such effects are more or less likely to affect conclusions.

## Appendix

### Proof of Proposition 2

As notational shorthand, define  $T_j = E[T^j] = E[S^j/2]$ , where  $S$  is the degree distribution of the network. Note that, because  $S$  can only take non-negative values,  $T_j$  is real-valued and non-negative for all  $j \geq 0$ . In this proof we shall make repeated use of the following result, which is an immediate consequence of Hölder's inequality.

**Proposition 9.** For any  $r > 1$ ,  $a \geq 0$ ,  $b \geq 0$ ,

$$T_{a+b} \leq T_{ar}^{\frac{1}{r}} T_{bs}^{\frac{1}{s}}$$

<sup>27</sup> See, for example, <https://www.ft.com/content/d3754014-30da-11e7-9555-23ef563ecf9a>.

where  $\frac{1}{r} + \frac{1}{s} = 1$ . Equality holds if and only if  $ar = bs$ , except in the trivial case when  $T$  is a constant.

Let us assume that Proposition 2 is false, and that there is some non-trivial network for which  $K^* \geq K^\dagger$ . We can write this as:

$$\begin{aligned} \frac{1}{4} \left( \frac{T_2}{T_1} + \frac{T_1}{T_2} \right)^2 &\geq \frac{1}{4} \left( \frac{T_4 - T_2^2 - T_2 + T_1^2}{T_3 - T_2 T_1} \right)^2 + 1 \\ \iff \left( \frac{T_2}{T_1} - \frac{T_1}{T_2} \right)^2 (T_3 - T_2 T_1)^2 &\geq (T_4 - T_2^2 - T_2 + T_1^2)^2 \end{aligned}$$

We can take square roots of both sides of this expression without changing the direction of the inequality, so long we are sure that the square root of each side is non-negative. The following statements ensure that it is indeed the case.

$$\begin{aligned} T_4 - T_2^2 - T_2 + T_1^2 &= \text{Var}[S] - \text{Var}[\sqrt{S}] \geq 0; \\ \frac{T_2}{T_1} - \frac{T_1}{T_2} &= \frac{E[S]}{E[\sqrt{S}]} - \frac{E[\sqrt{S}]}{E[S]} \geq 0; \text{ and} \\ T_3 - T_2 T_1 &= E[T^3] - E[T^2]E[T] \geq 0. \end{aligned}$$

The first two are obviously true, because  $S \geq 1$  with probability 1. The third can be proved using Proposition 9 with  $a = 2$ ,  $b = 0$ ,  $r = \frac{3}{2}$ ,  $s = 3$ .

That means that we can take square roots of both sides of the inequality above to obtain:

$$\begin{aligned} \left( \frac{T_2}{T_1} - \frac{T_1}{T_2} \right) (T_3 - T_2 T_1) &\geq T_4 - T_2^2 - T_2 + T_1^2 \\ \iff \left( \frac{T_2}{T_1} - \frac{T_1}{T_2} \right) T_3 - T_2^2 + T_1^2 &\geq T_4 - T_2^2 - T_2 + T_1^2 \\ \iff \left( \frac{T_2}{T_1} - \frac{T_1}{T_2} \right) T_3 &\geq T_4 - T_2 \\ \iff T_2^2 T_3 + T_1 T_2^2 &\geq T_1 T_2 T_4 + T_1^2 T_3. \end{aligned}$$

Now we shall use Proposition 9 to show that this is not true for any non-trivial network, and thus obtain a contradiction.

Setting  $a = \frac{4}{3}$ ,  $b = \frac{2}{3}$ ,  $r = 3$ ,  $s = \frac{3}{2}$ , we have  $T_2 < T_4^{\frac{1}{3}} T_1^{\frac{2}{3}}$ . Setting  $a = \frac{8}{3}$ ,  $b = \frac{1}{3}$ ,  $r = \frac{3}{2}$ ,  $s = 3$ , we have  $T_3 < T_4^{\frac{2}{3}} T_1^{\frac{1}{3}}$ . Multiply these together gives  $T_2 T_3 < T_1 T_4$ , and so  $T_2^2 T_3 < T_1 T_2 T_4$ .

Setting  $a = \frac{3}{2}$ ,  $b = \frac{1}{2}$ ,  $r = 2$ ,  $s = 2$  in Proposition 9, we can show  $T_2^2 < T_1 T_3$ . This means  $T_1 T_2^2 < T_1^2 T_3$ . Adding these up gives  $T_2^2 T_3 + T_1 T_2^2 < T_1 T_2 T_4 + T_1^2 T_3$ , and we have our contradiction.

We have shown that  $K^* \geq K^\dagger$  can only be true for a trivial network. Thus, the result is proved.  $\square$

### Proof of Proposition 3

#### Bounds for $K^*$

For  $K^*$ , the upper bound  $\frac{N^2}{4(N-1)}$  is achieved if  $P(S = N-1) = 1$ , that is, if we have a Duffie and Zhu network, and the lower bound of 1 is achieved if  $S$  can only take values on 0 or 1.

Let us write  $K^*(x) = \frac{1}{4} \left( x + \frac{1}{x} \right)^2$  where  $x = \frac{E[S]}{E[\sqrt{S}]}$ . In the feasible region  $x \geq 1$ ,  $K^*$  has a minimum at  $x = 1$  and is increasing for  $x > 1$ . Thus the lower bound for  $K^*$  is 1. This is attained when  $x = 1$ ; i.e.  $E[S] = E[\sqrt{S}]$ , which implies that  $S$  only takes values on 0 and 1.

Now consider the upper bound. Suppose there is some  $S$  for which  $K^*(x) \geq \frac{N^2}{4(N-1)}$ . Then, since  $K^*(x)$  is an increasing function,  $x \geq \sqrt{N-1}$ . That means that  $E[S] \geq \sqrt{N-1} E[\sqrt{S}]$ , which can only be true if  $P(S = N-1) = 1$ ; that is, we have the Duffie and Zhu network.  $\square$

### Bounds for $K^\dagger$

For  $K^\dagger$ , the lower bound 1 is achieved when  $\text{Var}[S] = \text{Var}[\sqrt{S}]$ ; i.e. when  $S$  only takes values on 0 or 1. It is easy to see from the definition of  $K^\dagger$  that it cannot attain a value lower than 1.

The proof for the upper bound is more complicated, so we first explain heuristically how it proceeds. We are trying to find a candidate probability vector  $p \in [0, 1]^N$  which maximises  $K^\dagger(p)$  subject to  $\sum_i p_i = 1$  (where  $i$  takes values in  $\{0, 1, \dots, N-1\}$ ). Proposition 3 stipulates that we are maximising over the set for which  $K^\dagger(p)$  is defined, so we can exclude any vectors  $p$  with any element with the value 1. We call these excluded points 'vertices'.

We will show that  $K^\dagger(p)$  is maximised on the boundary of this feasible set. That means we reduce our focus from a space with  $N$  dimensions to one with  $N-1$  dimensions. By the same argument  $K^\dagger(p)$  must be maximised on the boundary of that set too which has  $N-2$  dimensions, and so on inductively. Eventually we are reduced to one-dimensional line segments; that is, the subspaces where  $p$  has at most two non-zero elements. At this point, we cannot employ the inductive argument any more, because the boundaries of these line segments are the vertices at which are not contained in the feasible set for  $p$ . Therefore we know  $K^\dagger(p)$  is maximised on these line segments, and we can then finish the proof by examining these to find the maximal value.

Write  $\Omega(p) = \frac{\text{Var}[S] - \text{Var}[\sqrt{S}]}{E[S^{1.5}] - E[S]E[\sqrt{S}]}$ . Since  $\Omega(p) \geq 0$ , maximising  $K^\dagger = \frac{1}{4}\Omega^2 + 1$  is equivalent to maximising  $\Omega(p)$ . We can write down the following Kuhn-Tucker conditions:

$$\frac{\partial \Omega}{\partial p_i} + \lambda_i + \zeta = 0, \quad \forall i \quad (28)$$

$$\lambda_i p_i = 0, \quad \forall i \quad (29)$$

$$\zeta \left( \sum_i p_i - 1 \right) = 0, \quad (30)$$

where each  $\lambda_i$  is the Lagrange multiplier for the constraint  $p_i \geq 0$ , and  $\zeta$  is the Lagrange multiplier for the constraint  $\sum_i p_i = 1$ . As we are trying to find the maximum of  $\Omega$ , the Lagrange multipliers are all non-negative.

Computing the partial derivative of  $\Omega$ , Eq. (28) becomes:

$$-(\lambda_i + \zeta)v^2 = (E_{1.5} - E_1 E_{0.5})(i^2 - 2iE_1 - i + 2\sqrt{i}E_{0.5}) - (E_2 - E_1^2 - E_1 + E_{0.5}^2)(i^{1.5} - iE_{0.5} - \sqrt{i}E_1), \quad (31)$$

where  $v$  is the denominator of  $\Omega$ , and  $E_m$  is shorthand for  $E[S^m]$ .

Multiplying (31) by  $p_i$  and summing over  $i$  gives:

$$-N\zeta v^2 = (E_{1.5} - E_1 E_{0.5})(E_2 - 2E_1^2 - E_1 + 2E_{0.5}^2) - (E_2 - E_1^2 - E_1 + E_{0.5}^2)(E_{1.5} - 2E_1 E_{0.5}) = E_1(E_{0.5}E_2 - E_1 E_{1.5}) + E_{0.5}(E_{0.5}E_{1.5} - E_1^2). \quad (32)$$

But all the terms on the right-hand side of (32) are greater than or equal to zero,<sup>28</sup> while those on the left-hand side are less than or equal to zero. Therefore each term must be exactly equal to zero, and so  $S$  is a constant. At such points,  $\Omega$  is not defined, so we can surmise that  $\Omega(p)$  is maximised on the boundary of the feasible set for  $p$ .

This boundary is the set of points in  $[0, 1]^N$  with at least one zero element. Suppose that  $\Omega(p)$  is maximised on a point on the boundary where the element in position  $i'$  is zero. Then we can try to maximise  $\Omega(p)$  on the corresponding subspace, which has  $N-1$  dimensions. Repeating the analysis above, we can see that  $\Omega$  must be maximised on the boundary of this subspace, which is the set

of points in  $[0, 1]^N$  with at least two zero elements, one of which is in position  $i'$ .

Repeating this argument  $N-2$  times we can infer that  $\Omega$  is maximised on the set of points in  $[0, 1]^N$  which have exactly two non-zero elements. We cannot go any further, because the boundary of this subspace is the set of points with only one non-zero element; that is, the vertices, which we know do not lie in the feasible set of  $p$ .

Consider a general point  $p$  in this subspace, with its non-zero elements at  $i$  and  $j$ , where  $i < j$ . Suppose  $p_i = \delta$  and  $p_j = 1 - \delta$ , for some  $0 < \delta < 1$ . Then  $E_m = \delta i^m + (1 - \delta)j^m$  for each  $m$ , and so:

$$\Omega(p) = \frac{(j-i)^2 - (\sqrt{j} - \sqrt{i})^2}{(j-i)(\sqrt{j} - \sqrt{i})} \quad (33)$$

with the terms in  $\delta$  cancelling out in the numerator and denominator of (33). This means that  $\Omega$  is constant along any line in the subspace, and we simply need to choose  $i$  and  $j$  to maximise (33).

Noting that  $j-i = (\sqrt{j} - \sqrt{i})(\sqrt{j} + \sqrt{i})$ , we can cancel terms to obtain:

$$\Omega(p) = \sqrt{j} + \sqrt{i} - \frac{1}{\sqrt{j} + \sqrt{i}}, \quad (34)$$

which is maximised when  $\sqrt{j} + \sqrt{i}$  is as large as possible. This means that  $j = N-1$  and  $i = N-2$ , and so  $\Omega(p) = 2\sqrt{N-2}$  and  $K^\dagger = N-1$ .  $\square$

### Proof of Proposition 4

We have already shown the second part is true, because  $\alpha = 3$  for the DMS network and so  $K^\dagger$  grows without bound according to Table 1. (In fact this will be true for any scale-free network.) Now we just need to show that the asymptotic value of  $K^*$  – which we know tends to a finite limit – is smaller than 2.

Observe that  $E_t(S) = \frac{4t-2}{t+1} \rightarrow 4$  in the DMS network, since at each step of the network construction process four additional links are created (each new node has an in-link and an out-link with two existing nodes). (Dorogovtsev et al., 2001) show that:

$$\lim_{t \rightarrow \infty} P_t(s) = \frac{12}{s(s+1)(s+2)},$$

and so the term  $\frac{E_t[S]}{E_t[\sqrt{S}]}$  is asymptotically equal to:

$$4 / \left( 12 \sum_{s=2}^{\infty} \frac{1}{\sqrt{s}(s+1)(s+2)} \right) = 2.17.$$

This gives us an asymptotic value for  $K^* = 1.73 < 2$ . Therefore in the infinite limit the CCP never reduces expected netted exposures, except in the trivial case where  $K = 1$ ; i.e. when the CCP clears the only asset class.  $\square$

### Proof of Proposition 5

Let us use  $\mu$  and  $V$  to denote  $E[S]$  and  $\text{Var}[S]$  respectively, for a given  $N$ . Then, as  $N \rightarrow \infty$ , we can make use of the following approximations:

$$E[\sqrt{S}] = \mu^{0.5} E \left[ 1 + \left( \frac{S}{\mu} - 1 \right) \right]^{0.5} \approx \mu^{0.5} \left[ 1 - \frac{V}{8\mu^2} \right], \quad (35)$$

and

$$E[S^{\frac{3}{2}}] = \mu^{1.5} E \left[ 1 + \left( \frac{S}{\mu} - 1 \right) \right]^{1.5} \approx \mu^{1.5} \left[ 1 + \frac{3V}{8\mu^2} \right]. \quad (36)$$

In both cases we have expanded the binomial series around 1 and neglected cubic and higher-order terms. Substituting these into our expressions for  $K^*$  and  $K^\dagger$ , we find that both  $K^* \sim O(\mu)$  and  $K^\dagger \sim O(\mu)$ .  $\square$

<sup>28</sup> We showed this in the proof of Proposition 2, where we used Proposition 9 to show that  $T_1 T_4 \geq T_2 T_3$  and  $T_1 T_3 \geq T_2^2$ , with equality if and only if  $T$  is a constant.

### Proof of Proposition 6

1. Let  $K_1^*$  and  $K_2^*$  represent the values of  $K^*$  for constellations  $\Psi_1$  and  $\Psi_2$  respectively, as given by equation (★'). We need to show that  $K_1^* < K_2^*$ . As  $A$  and the distribution  $S$  are the same under both constellations, we can show that:

$$\begin{aligned} K_1^* &< K_2^*, \\ &\Leftrightarrow \frac{1}{A} \left( \sqrt{K_1^*} + \sqrt{K_1^* - A} \right) < \frac{1}{A} \left( \sqrt{K_2^*} + \sqrt{K_2^* - A} \right), \\ &\Leftrightarrow \left( \sum_{m=1}^M \sqrt{a_m} \right)^{-1} \cdot \frac{E[S]}{E[\sqrt{S}]} < \left( \sqrt{a_1 + a_2} + \sum_{m=3}^M \sqrt{a_m} \right)^{-1} \cdot \frac{E[S]}{E[\sqrt{S}]}, \\ &\Leftrightarrow \sqrt{a_1} + \sqrt{a_2} > \sqrt{a_1 + a_2}, \end{aligned}$$

which is true for any  $a_1, a_2 > 0$ .

Now let us define  $K_1^\dagger$  and  $K_2^\dagger$  similarly. We have:

$$\begin{aligned} K_1^\dagger &< K_2^\dagger, \\ &\Leftrightarrow \frac{A \text{Var}[S] - \left( \sum_{m=1}^M \sqrt{a_m} \right)^2 \text{Var}[\sqrt{S}]}{\left( \sum_{m=1}^M \sqrt{a_m} \right) \left( E[S^{\frac{3}{2}}] - E[S]E[\sqrt{S}] \right)} \\ &< \frac{A \text{Var}[S] - \left( \sqrt{a_1 + a_2} + \sum_{m=3}^M \sqrt{a_m} \right)^2 \text{Var}[\sqrt{S}]}{\left( \sqrt{a_1 + a_2} + \sum_{m=3}^M \sqrt{a_m} \right) \left( E[S^{\frac{3}{2}}] - E[S]E[\sqrt{S}] \right)}, \\ &\Leftrightarrow \frac{A \text{Var}[S]}{\sum_{m=1}^M \sqrt{a_m}} - \left( \sum_{m=1}^M \sqrt{a_m} \right) \text{Var}[\sqrt{S}] \\ &< \frac{A \text{Var}[S]}{\sqrt{a_1 + a_2} + \sum_{m=3}^M \sqrt{a_m}} - \left( \sqrt{a_1 + a_2} + \sum_{m=3}^M \sqrt{a_m} \right) \text{Var}[\sqrt{S}], \\ &\Leftrightarrow 0 < A \text{Var}[S] \left( \frac{1}{\sqrt{a_1 + a_2} + \sum_{m=3}^M \sqrt{a_m}} - \frac{1}{\sum_{m=1}^M \sqrt{a_m}} \right) \\ &+ \left( \sqrt{a_1} + \sqrt{a_2} - \sqrt{a_1 + a_2} \right) \text{Var}[\sqrt{S}], \end{aligned} \quad (37)$$

which is always true since  $\sqrt{a_1} + \sqrt{a_2} > \sqrt{a_1 + a_2}$  for any  $a_1, a_2 > 0$ . Thus the first part of the proposition is proved.

2. Let  $K_2^*$  and  $K_3^*$  represent the values of  $K^*$  for constellations  $\Psi_2$  and  $\Psi_3$  respectively. We need to show that  $K_2^* > K_3^*$ . We have:

$$\begin{aligned} K_2^* &> K_3^*, \\ &\Leftrightarrow \frac{1}{A+1} \left( \sqrt{K_2^*} + \sqrt{K_2^* - (A+1)} \right) \\ &> \frac{1}{A+1} \left( \sqrt{K_3^*} + \sqrt{K_3^* - (A+1)} \right), \\ &\Leftrightarrow \left( \sqrt{a_1 + 1} + \sum_{m=2}^M \sqrt{a_m} \right)^{-1} \cdot \frac{E[S]}{E[\sqrt{S}]} \\ &> \left( \sqrt{a_2 + 1} + \sum_{m \neq 2}^M \sqrt{a_m} \right)^{-1} \cdot \frac{E[S]}{E[\sqrt{S}]}, \\ &\Leftrightarrow \sqrt{a_1 + 1} + \sqrt{a_2} < \sqrt{a_1} + \sqrt{a_2 + 1}, \\ &\Leftrightarrow \sqrt{a_1 + 1} - \sqrt{a_1} < \sqrt{a_2 + 1} - \sqrt{a_2}, \end{aligned} \quad (38)$$

which is true since  $a_1 > a_2$  and  $\sqrt{X+1} - \sqrt{X}$  is a decreasing function.

Now let us define  $K_2^\dagger$  and  $K_3^\dagger$  similarly. We have:

$$K_2^\dagger > K_3^\dagger,$$

$$\begin{aligned} &\Leftrightarrow \frac{(A+1) \text{Var}[S] - \left( \sqrt{a_1 + 1} + \sum_{m=2}^M \sqrt{a_m} \right)^2 \text{Var}[\sqrt{S}]}{\left( \sqrt{a_1 + 1} + \sum_{m=2}^M \sqrt{a_m} \right) \left( E[S^{\frac{3}{2}}] - E[S]E[\sqrt{S}] \right)} > \\ &\frac{(A+1) \text{Var}[S] - \left( \sqrt{a_2 + 1} + \sum_{m \neq 2}^M \sqrt{a_m} \right)^2 \text{Var}[\sqrt{S}]}{\left( \sqrt{a_2 + 1} + \sum_{m \neq 2}^M \sqrt{a_m} \right) \left( E[S^{\frac{3}{2}}] - E[S]E[\sqrt{S}] \right)}, \\ &\Leftrightarrow (A+1) \text{Var}[S] \left( \frac{1}{\sqrt{a_1 + 1} + \sqrt{a_2} + \sum_{m=3}^M \sqrt{a_m}} \right. \\ &\quad \left. - \frac{1}{\sqrt{a_1} + \sqrt{a_2 + 1} + \sum_{m=3}^M \sqrt{a_m}} \right) \\ &\quad + \text{Var}[\sqrt{S}] \left( \sqrt{a_1} + \sqrt{a_2 + 1} - \sqrt{a_1 + 1} - \sqrt{a_2} \right) > 0, \end{aligned} \quad (39)$$

which is always true since  $a_1 > a_2$ . Thus all parts of the proposition are proved.  $\square$

### Proof of Proposition 8

Consider a single agent  $i$  and a single asset class  $k$ . In the trading network, the agent has  $S_i$  links, each distributed  $N(0, \sigma^2)$ . As these are all exposures to the same asset class, they are perfectly correlated and so the total market risk conditional on  $S_i$  is  $M_{ik}|S_i \sim N(0, \sigma^2 S_i^2)$ . Then market risk is:

$$\begin{aligned} \text{Var}[M_{ik}] &= E[\text{Var}[M_{ik}|S_i]] + \text{Var}[E[M_{ik}|S_i]], \\ &= E[\sigma^2 S_i^2] + \text{Var}[0] = \sigma^2 E[S_i^2]. \end{aligned} \quad (40)$$

For the clearing network, exactly the same procedure gives market risk of  $\tilde{\sigma}^2 E[\tilde{S}^2]$  for each agent in each asset class. As the number of agents and asset classes does not change, conservation of market risk means that the two terms must be equal.

The assumption of perfect correlation assumes that the agent has no offsetting derivative positions. We can allow for this, so long as the extent of such hedging is the same in the trading and clearing networks. Let  $h_{ik}$  be the proportion of positions in asset class  $k$  which do not represent hedging positions. Then overall market risk in the trading network is  $h_{ik} \sigma^2 E[S_i^2]$  and market risk in the clearing network is  $h_{ik} \tilde{\sigma}^2 E[\tilde{S}^2]$ , so Proposition 8 still holds.

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